Reflective Essay Number One:
The Universe is a Temple,
Life is a Spiral,
God is a Mystery

David Ernest Wächter

What a wonderful title!

"Look, between the stones is a blade of grass;
And all the rites of the high Mysteries,
And the runes of all witcheries,
Are written upon it" (Valiente 12).

"When we look into the heart of a flower, we see clouds, sunshine, minerals, time, the earth, and everything else in the cosmos in it" (Nhat Hanh 11).

"'Everything is a leaf' -- Goethe" (Conway 114).

Preface

Although this book is intended primarily for readers in their early teens, it is my hope that there is something of value here for persons of all ages. Younger children will benefit from the reading company of an adult, especially in performing some of the experiments.

The Temple-Universe

The universe is a temple, the holy living temple where God sings and dances. God's melodious breath and dancing steps are moving everywhere throughout this magic temple, filling the universe with beautiful and diverse patterns of life. You are a priest in this holy temple, called by God to care for all of this wondrous creation. An important part of caring for creation involves listening to its many mysteries. There is perhaps no greater mystery in our universe than the mystery of number. Everything in the universe is based upon number. Number is like a tool used by God in building this temple-universe. Number precedes and patterns all of creation. By studying the patterns and the breaks in patterns observable in nature we can enter into the mind of God. We are thereby able to become better caretakers of creation and to live fuller, more harmonious lives.

wonderful line!

Fascinating point of view!

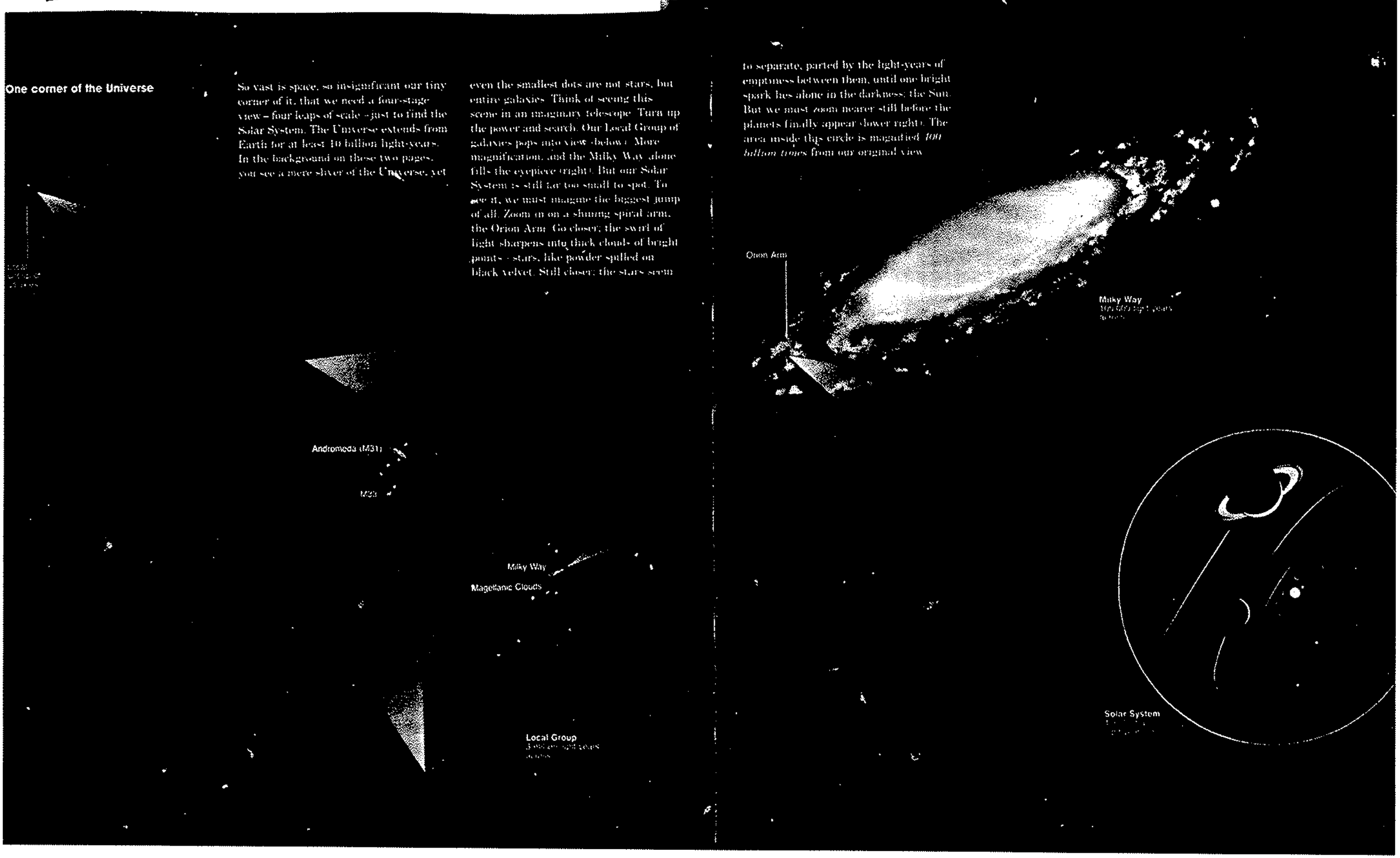
lovely!

God's gift of life can be thought of as arising from the divine pattern at work in matter. In the story of Genesis God forms Adam out of moist earth (or matter). But Adam only becomes a living soul when the pattern of divine spirit is breathed by God into this virgin matter. One can think of pattern and matter as our father and mother in a sense. The English word "pattern" comes from the Latin word "pater" which means "father". The English word "matter" comes from the Latin word "mater" which means "mother". We need both the pattern and the matter to be what we are. ✓ There are countless types of mathematical patterns in matter, everyone of which reveals some

Wachter 1a

One corner of the Universe

So vast is space, so insignificant our tiny corner of it, that we need a four-stage view – four leaps of scale – just to find the Solar System. The Universe extends from Earth for at least 10 billion light-years. In the background on these two pages, you see a mere sliver of the Universe, yet even the smallest dots are not stars, but entire galaxies. Think of seeing this scene in an imaginary telescope. Turn up the power and search. Our Local Group of galaxies pops into view (below). More magnification, and the Milky Way alone fills the eyepiece (right). But our Solar System is still far too small to spot. To see it, we must imagine the biggest jump of all. Zoom in on a shining spiral arm, the Orion Arm. Go closer; the swirl of light sharpens into thick clouds of bright points – stars, like powder spilled on black velvet. Still closer: the stars seem to separate, parted by the light-years of emptiness between them, until one bright spark lies alone in the darkness: the Sun. But we must zoom nearer still before the planets finally appear (lower right). The area inside this circle is magnified *100 billion times* from our original view.



deep mystery about life and about God. We shall take a look at one specific pattern in matter, the spiral, and explore how this pattern turns up again and again in the material universe, reflecting upon the spiritual significance of this mathematical pattern.

Great way to frame your discussion. I, too, have thought about this a lot.

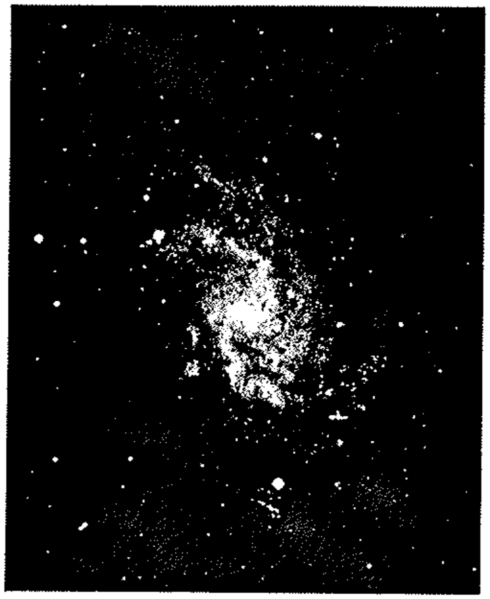
The spiral of life

Life is a spiral, an open-ended circling, an evolving cycle of endless expansion, a growing progression of constant twists and turns. Life began on Earth with simple one celled bacteria called archae, or "ancient ones", floating in the ocean. Now, today, after four billion years of spiraling diversification and evolution, a mind boggling vast array of plant and animal species populates our planet and strange animals with self-awareness called humans have appeared. If life always stayed the same it would be a boring straight line. Thankfully life is a beautifully creative spiral. Just as a spiral is always growing larger and larger, life in the universe is ever growing more complex and evolving towards higher and higher levels of consciousness. And like a spiral, our material universe itself is ever expanding farther and faster. Whenever you see aspects of a spiral somewhere in nature, you are observing powerful life forces at work.

The spiral galaxy

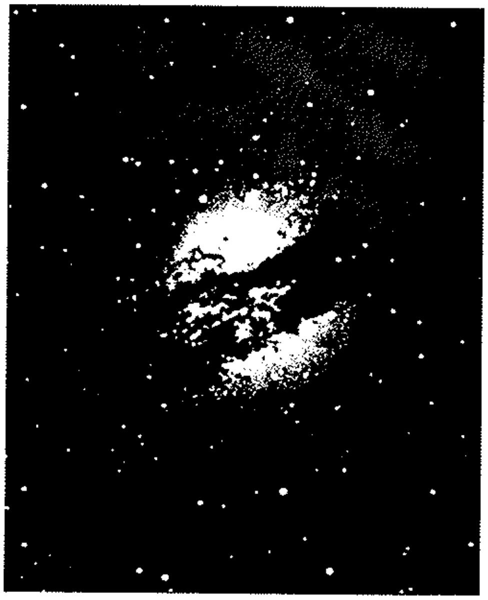
← These subheadings are an effective organizational tool, especially when dealing with a complex subject like this. - very helpful

We live in a spiral galaxy. A galaxy is a vast cluster of billions of stars. Depending upon their shape, galaxies are classified as either spiral, elliptic or irregular. Our home galaxy, the Milky Way, is spiral shaped. Our Sun, Earth

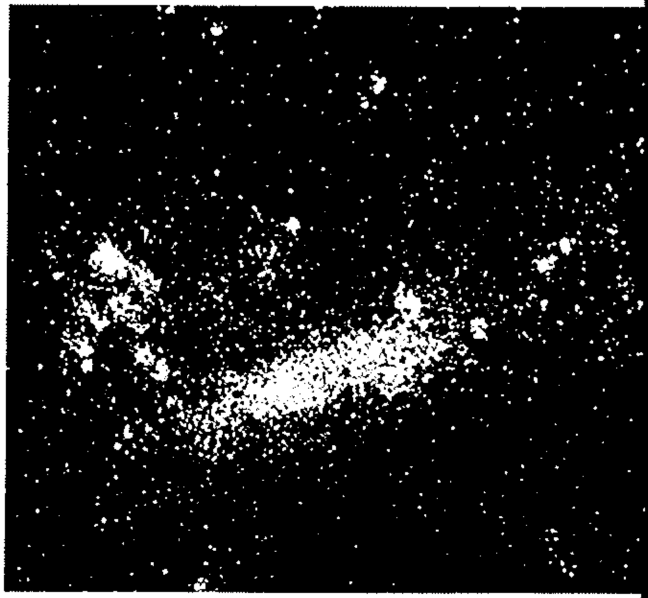


M33 Spiral

NGC 1300 Barred spiral



NGC 5128 Elliptical (peculiar)



Large Magellanic Cloud Irregular

and entire solar system are located at the outer edge of one of the Milky Way's spiral arms. If you go outside and find a dark place where you can look up at the night sky, you will see a lighter, whiter region in the center of the sky that looks like a river of milk running through the heavens. This is the view of the Milky Way from planet Earth. It is the light from all the billions of stars in our galaxy that gives this lighter region of the night sky its milky appearance. Take a look at this region of the sky through a telescope or binoculars if you have them and see how many stars you can see. Now find the constellation Sagittarius in the night sky. You may need to use a star chart or get the help of an adult to help you locate the various star constellations. Gazing at Sagittarius you are gazing into the center of our galaxy. It is around this constellation that all the three hundred billion stars of the Milky Way revolve.

If you use a telescope or binoculars you can also see the Andromeda Galaxy, a neighbor to our Milky Way located 2.5 million light years away. Focus your telescope or binoculars upon the faint blur of light within the Andromeda constellation and you will see that the Andromeda Galaxy is also a spinning spiral.

The spiraling planets

We are spiraling though space. The planets revolve around the sun in an elliptic orbit. But because the sun is also moving through space, planetary orbits actually form elliptic spirals. If we were to represent the motion of the sun as a straight line then the motion of the Earth would be a spiral winding

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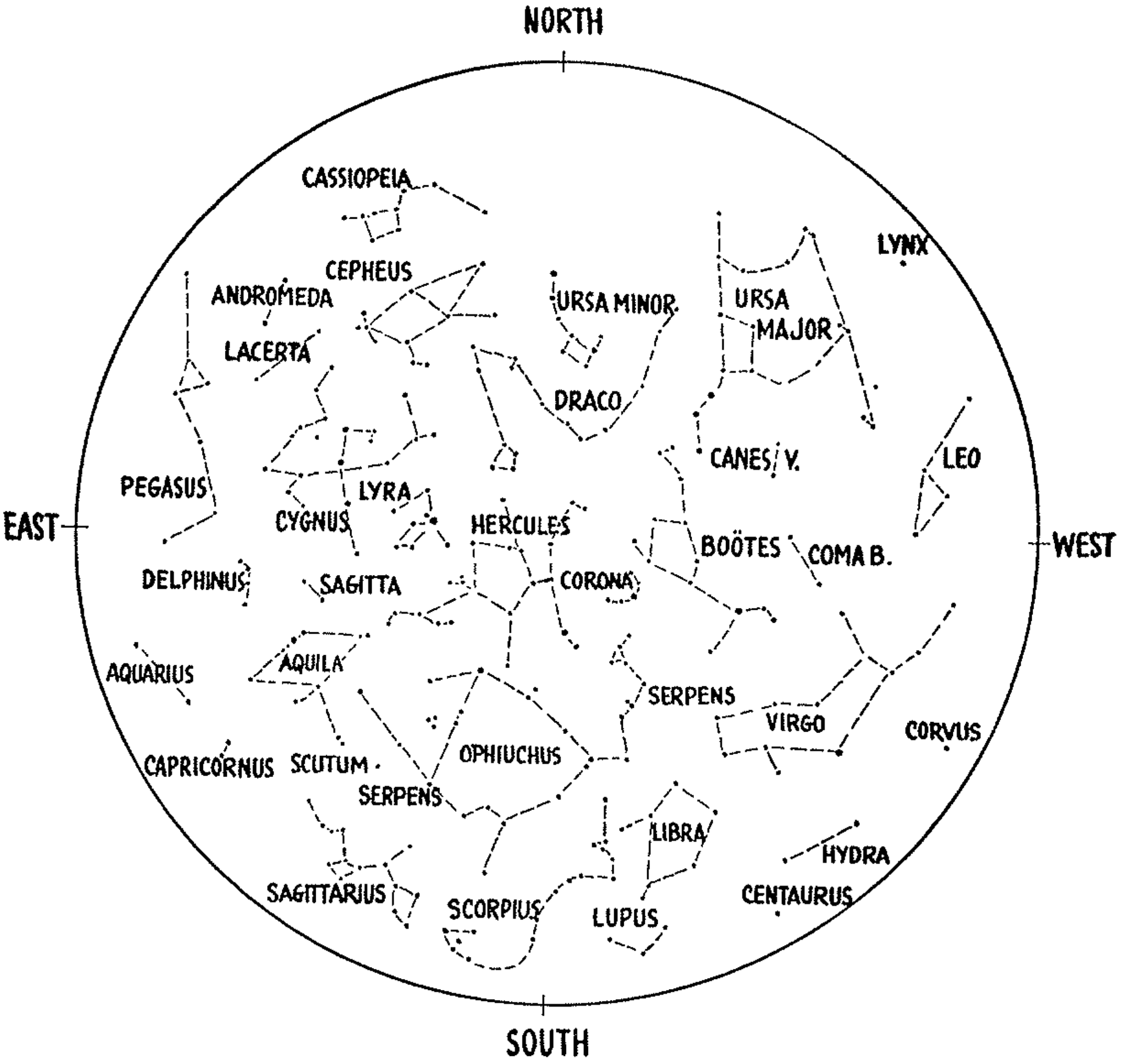
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WEST
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CORONA
AQUILA
AQUARIUS
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VIRGO
CORVUS
OPHIUCHUS
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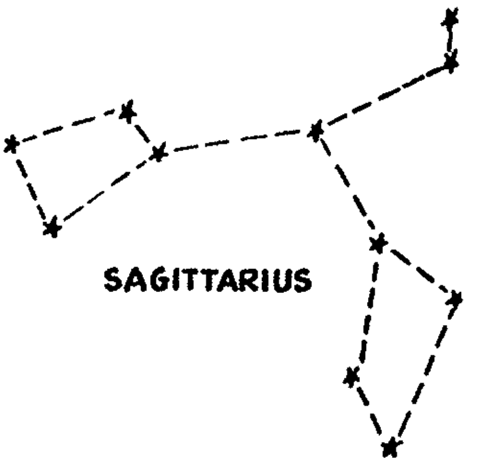
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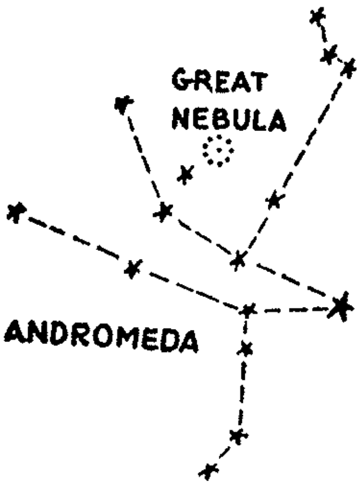

GREAT
NEBULA
ANDROMEDA

around this line. The moon's path would be a spiral winding around the spiral path of the Earth. Every year the moon spirals about three centimeters farther away from the Earth. Imagine all the spiraling paths of all the planets and satelittes spinning around our solar system.

For our next experiment find a yo-yo. Imagine that you are the sun and that the yo-yo at the end of the string is a planet. Now hold the string above your head and swing the yo-yo around your body in a circular motion while you walk up a hill. This imitates the spiral motion that a planet makes as it orbits a moving star.

good illustration

Curved Space Spiraling

The very space that makes up our universe is curved. Here is an experiment to help illustrate this concept. Get a bed sheet, four folding chairs, four books, a large orange or grapefruit and a small ball. Now place the four chairs out in a large square or rectangle and place a corner of the bedsheet upon the seat of each chair, holding the sheet down with a heavy book. Put the large orange or grapefruit in the center of the sheet. Imagine that the sheet is space and that the fruit is a celestial body such as a star or planet. The heavier the fruit, or celestial body, the more the sheet, or space, will become curved around that body. Next, take your small ball and roll it straight forward from one chair to one of the other two chairs directly across from and at a right angle to it. What happens to the ball? It should spiral around and eventually collide into the fruit. This is what happens when a meteor

(Wouldn't gravity affect the results of this?)

or other object in space comes too close to a star or planet. The gravity, or curved space, around the star or planet causes the straight traveling meteor to spiral towards it and either collide with it or burn up in its atmosphere.

Body spirals

We are marked by spirals. Not only do we find spirals in the formation of and motion of heavenly bodies, but we also find spirals in the structure of our own human bodies. In fact, spirals are responsible for the uniqueness of each human body. Did you know that the spirals in your fingerprints make you unique from every other person that ever lived? It's true. Everybody has spirals in their fingerprints but nobody has the exact same fingerprints. Get an ink pad or a marker and some paper. Cover the tips of your fingers in wet ink and press your fingertips on the paper. Look at the fingerprints you have made. How many spirals can you find?

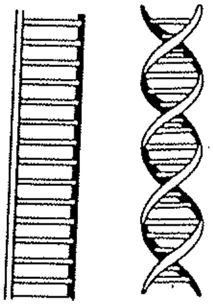
I never thought of this before!

Like fingerprints, DNA is made of spirals. And, just like your fingerprints, your DNA is different from everyone else's DNA who ever lived. DNA stands for deoxyribonucleic acid and is the blueprint for life found within every cell of the body. DNA consists of two interlocking spirals called a double helix. The information contained between these two spirals is what makes every living creature unique.

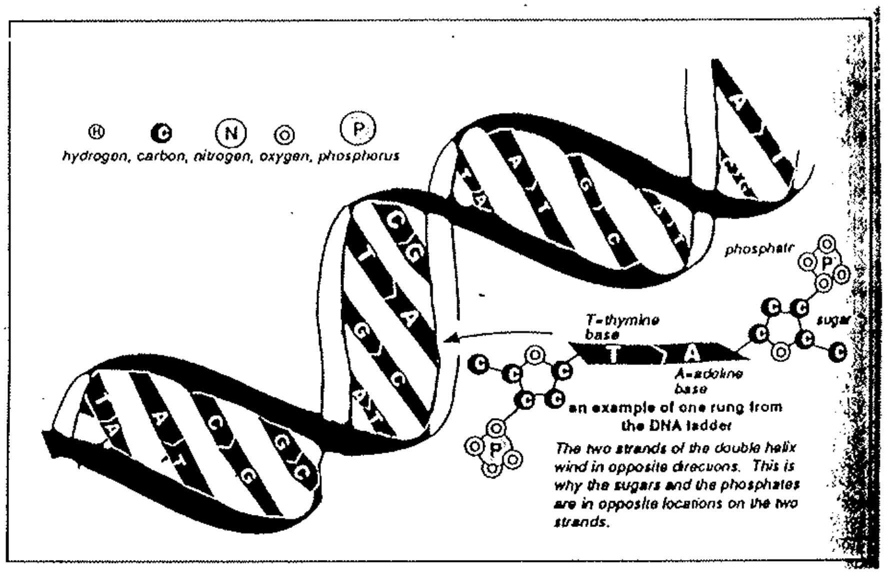
Spiral growth

I didn't know this!

We grow in spirals. Each hair and bone of your body grows in an equiangular spiral pattern. The equiangular spiral is a common growth pattern in nature, probably because such spirals

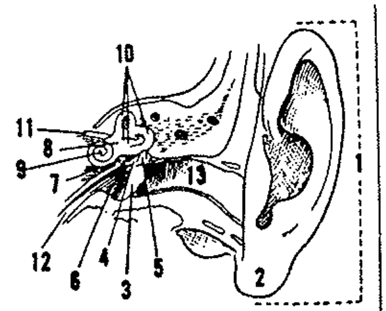


DNA is made from two helical strands that are joined at their bases to form a double helix.



an example of one rung from the DNA ladder

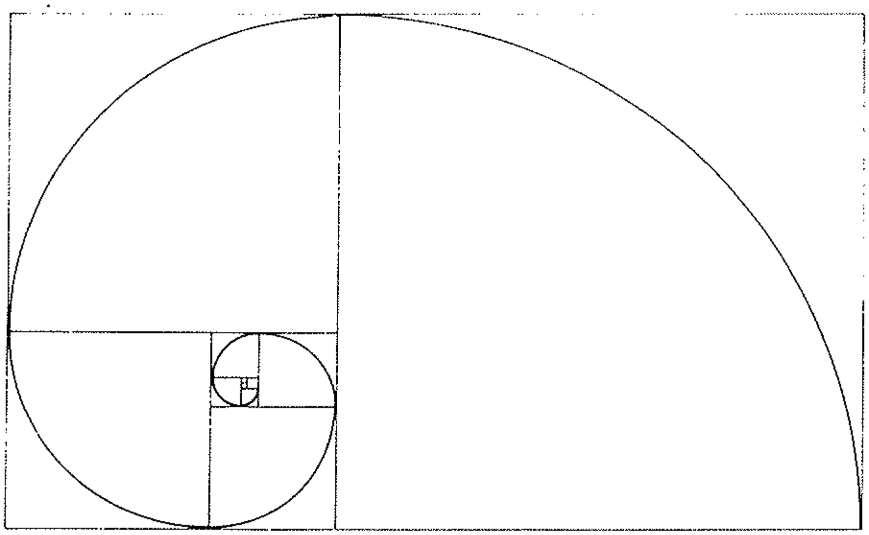
The two strands of the double helix wind in opposite directions. This is why the sugars and the phosphates are in opposite locations on the two strands.



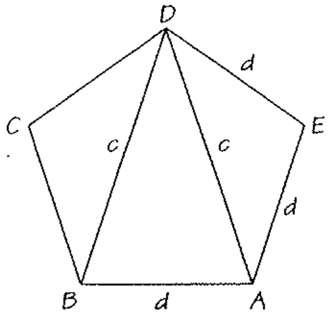
ear: *1* pinna, *2* lobe, *3* tympanic membrane, *4* incus, *5* malleus, *6* tympanum, *7* stapes, *8* vestibule, *9* cochlea, *10* semicircular canals, *11* auditory nerve, *12* eustachian tube, *13* auditory meatus retain a constant shape as they grow. Another equiangular spiral at work in the human body is the cochlea, a tiny spiral bone in your inner ear which allows you to hear. Another example of an equiangular spiral is the umbilical cord that provides nourishment for a baby growing in a mother's womb. There is even a spiral pattern to the electrical impulses which keep your heart beating.

The proportions of your body also contain a spiral pattern, one which you can actually measure. Get a tape measure. First measure your total height in either inches or centimeters. Second, measure from your toes to your belly-button. Third, use a calculator to divide your total height by the height to your belly-button. The number which you get should be close to 1.6180339887494..., known as the golden number or golden mean because it is the mathematical ratio of a special type of equiangular spiral called the golden spiral. The exact value of the golden mean is expressed mathematically by the equation $(1 + \sqrt{5})/2$, which represents the ratio of a regular pentagon's diagonal length divided by the length of its side.

The golden spiral is a common growth pattern not only in the human body, but throughout all of nature. Plants also grow in equiangular spirals. Find a plant with a long stem. Notice the rising spiral distribution of the leaves on the stem. Pick a leaf near the bottom of the stem to be your base leaf. Now starting at but not counting your base leaf, count the number of leaves above until you reach the next leaf located directly over your base leaf. The number of leaves that make up one



[redacted] The golden section rectangle with sides in the proportion 1 : ϕ or 1 : 1.61803 . . . increases or diminishes in size by means of the square. When the area of the square on its shorter side is deducted from this figure, the remainder is found to have the same proportions as the original rectangle. The resulting ϕ spiral is one of the common growth patterns in nature.



if
d = 1 then

c = 1.618

spiral turn on the plant's stem will likely be a number from the Fibonacci sequence (1,1,2,3,5,8,13,21,34,55,89...). Interestingly, the number of petals on most types of flowers will also be a number in the Fibonacci sequence. A lily has three petals. A geranium has five petals. A delphinium has eight petals. A marigold has thirteen petals. A daisy has either thirty-four, forty-five or eighty-nine petals. If the number of petals does not add up to one of these Fibonacci numbers, the number will either be a number twice as large as one of these Fibanacci numbers (2,4,6,10,16,26,42...) or a number from the Fibanacci sequence which begins with the numbers four and seven instead of one and one (4,7,11,18,29,47...). There are some species of mountain trees in which the Fibonacci sequence can be seen throughout the tree's entire sturcture, in the organization of its leaves, branches and roots.

Do you see the pattern in the Fibonacci sequence? To get the next number in the sequence you add the previous two numbers together. And the more you expand this sequence, the closer the last two numbers in the sequence will approximate the golden mean (1.618) when the last number is divided by the second to last number:

1/1 = 1

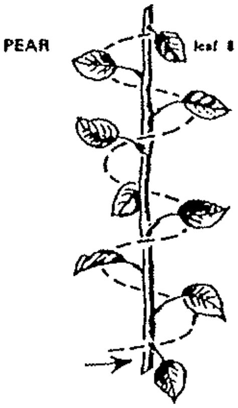
2/1 = 2

3/2 = 1.5

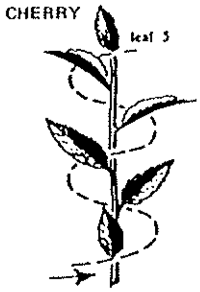
5/3 = 1.666

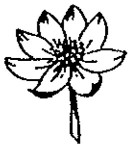
8/5 = 1.6

13/8 = 1.625

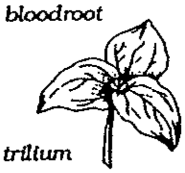


•*Arrangement of leaves, twigs and stems* is known as **phyllotaxis.** Select a leaf on a stem and count the number of leaves (assuming none have been broken-off) until you reach one directly in line with the one you selected. The total number of leaves (not counting the first one you selected) is usually a Fibonacci number in many plants, such as in elm, cherry or pear trees.

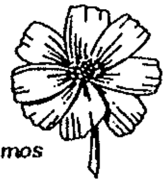




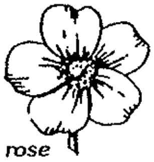
bloodroot



trillium



cosmos



wild rose

21/13 = 1.615

34/21 = 1.619

55/34 = 1.617

89/55 = 1.618

In fact, this division pattern holds true for any two numbers you start with. Pick any two numbers. Now add the two numbers together to get the third, add the second and third numbers to get the fourth, and so on until you have a long series of numbers. The more you expand this sequence the closer the division of the last number by the second to last number will approximate the number of the golden spiral, 1.618. Strange!

Next, find a pineapple and look at its base. Count the number of clockwise spirals and counterclockwise spirals that you see. You should find that a pineapple has eight rows of scales spiraling left and thirteen rows of scales spiraling right. Notice that 8 and 13 are consecutive Fibonacci numbers. If you look at the base of a pinecone you will also see clockwise and counterclockwise spirals. The number of clockwise and counterclockwise spirals will also be consecutive Fibonacci numbers. The same is true for sunflower seedheads and the seedheads of other flowers. Depending on the species of sunflower, the number of clockwise and counterclockwise spirals will be either 34 and 55, 55 or 89, or 89 and 144, all of which are consecutive Fibonacci numbers. As we already learned, when two consecutive Fibonacci numbers are divided, they approximate the ratio of the golden spiral (1.618). Seedheads make use of the golden mean in their structure because such a proportion

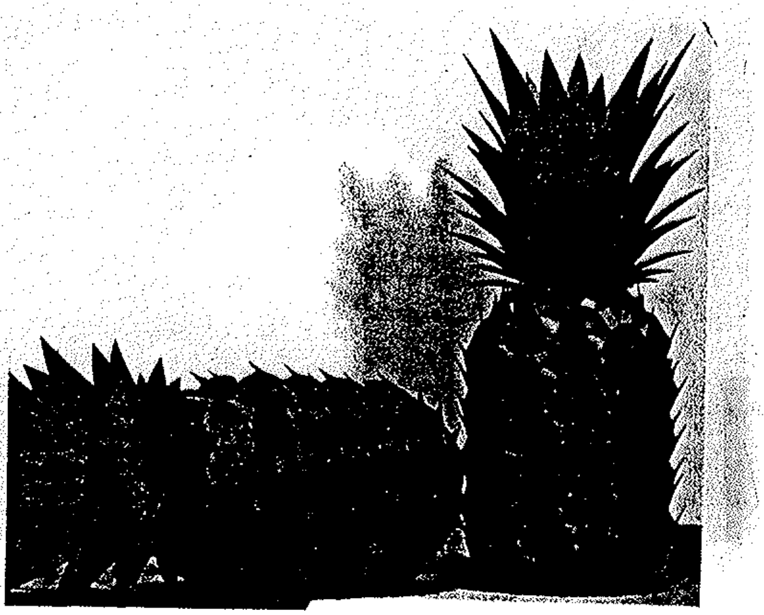


FIGURE 4.26 *Spirals on pineapples. (Photo courtesy of D.R. Fowler and A. Snider.)*

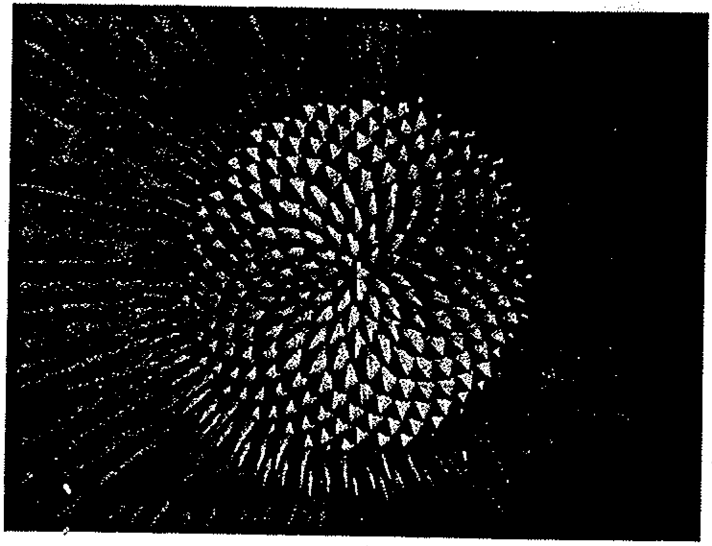
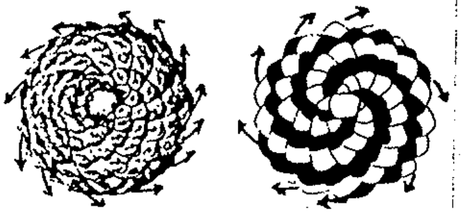
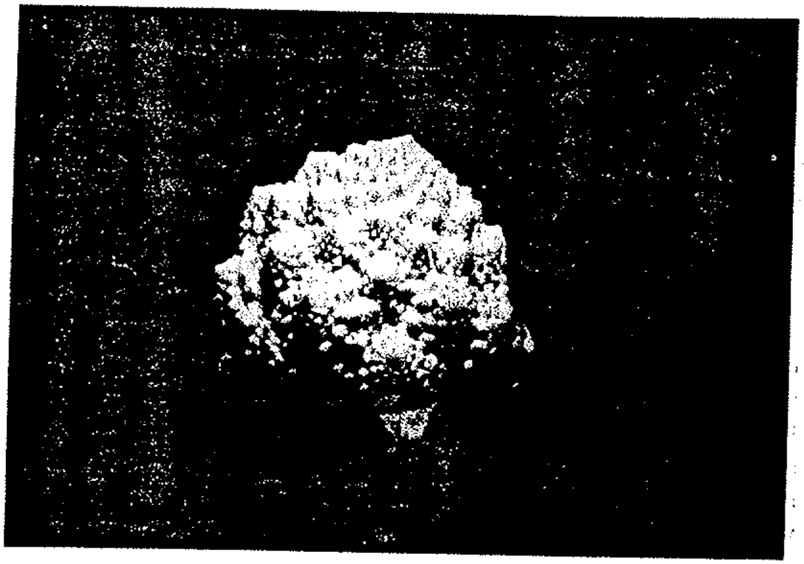


FIGURE 4.27 *Daisy capitulum. (Computer generated by Deborah R. Fowler on an IRIS workstation using an L-grammar algorithm.)*

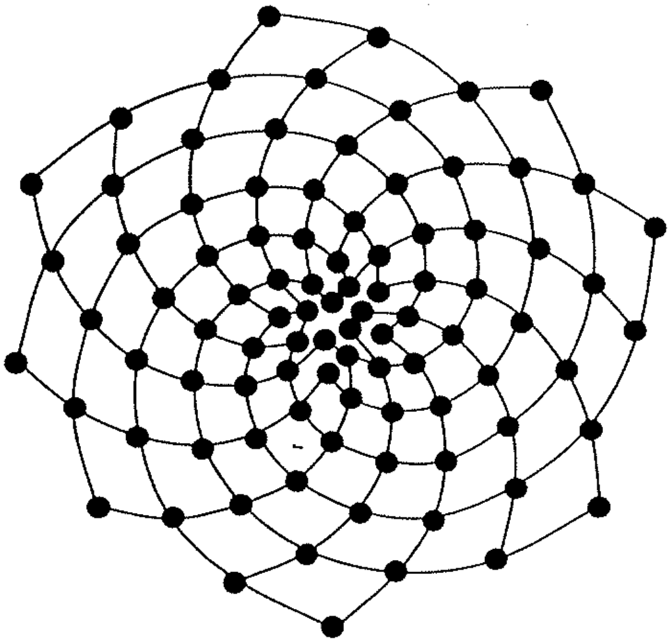


• *The pine cone numbers:* If the left and right handed spirals on a pine cone are counted, the two numbers are very often consecutive Fibonacci numbers. This also holds true for sunflower seedheads and seedheads of other flowers. The same is true of pineapples. Looking at the base of a pineapple count the number of left and right spirals composed of hexagonal shapes scales. They should be consecutive Fibonacci numbers.

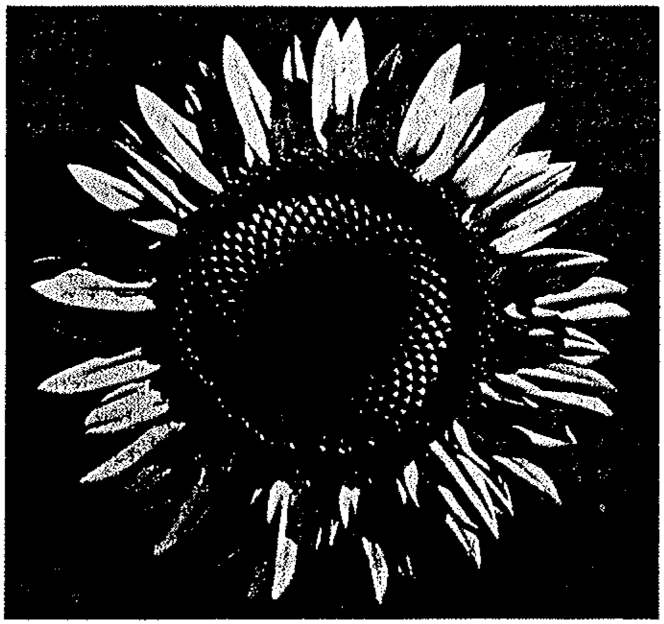


The inflorescence of a cauliflower. (Photo courtesy of E. Thirian.)

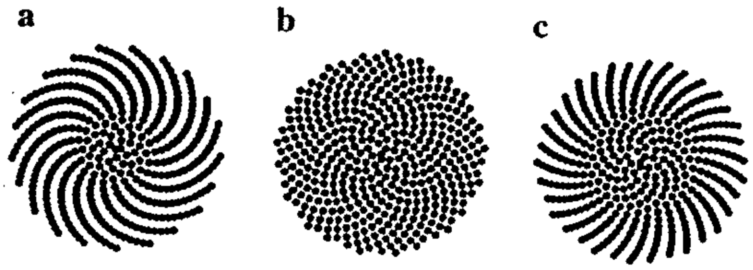
Spiral patterns in sunflowers.



[redacted]
Successive dots arranged at angles of 137.5° to each other along a tightly wound spiral (not shown) naturally fall into two families of loosely wound spirals that are immediately apparent to the eye. Here there are 8 spirals in one direction and 13 in the other—consecutive Fibonacci numbers.



Sunflower. (Courtesy of D.R. Fowler and P. Prusinkiewicz.)



[redacted] Packing primordia at angles (a) slightly less than the golden angle, (b) equal to the golden angle, (c) slightly more than the golden angle. Only in case (b) do they pack efficiently.

creates the most solid and compact seedhead possible.

More spirals in nature

Spirals appear throughout the natural world. Sprials can be seen in weather patterns, such as hurricanes and tornadoes, and in water patterns, such as whirlpools. Many shells, such as snail shells, nautilus shells, and other sea shells have a spiral shape. Many plants, such as fiddlehead ferns, caulifolower florets, and certain cacti and vines, have a very noticeable sprial shape. Among animals, spirals can be found in ram horns, certain goat antlers, orb spider webs and slime mould amoeba colonies. The spiral shape is even programmed into the brain of the mud wasp, which constructs its egg chamber entrance tube in the form of a spiral, thereby preventing invasion by larger insect predators. The involute curve is another spiral-like shape which can be found in nature, in eagle beaks, in shark dorsal fins, and in hanging palm leaves.

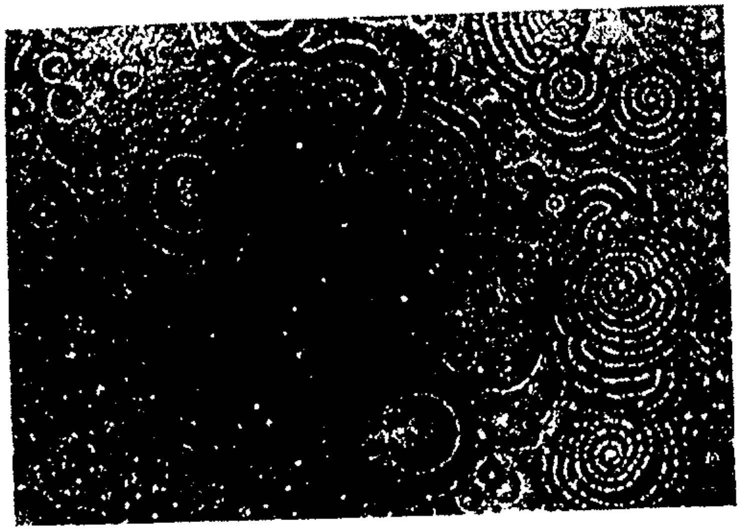
([illegible])

BZ reaction spirals

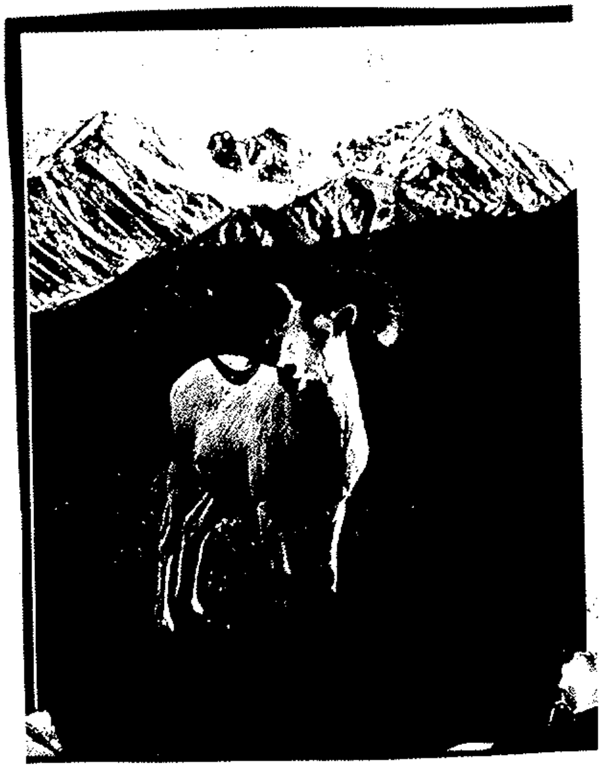
The following experiment, known as the Belousov-Zhabotinskii reaction, is an example of spontaneous rotating spiral formation. Make sure you have the assistance of an adult when conducting this experiment and perform it in a well ventilated room. You should be able to find the necessary chemicals in any school laboratory or from any chemical supply company, such as Antec, Inc. (800-448-2954). First, perform these three steps by mixing the following chemicals in separate glass beakers:

A) Dissolve 25 g sodium bromate in 335 ml of water, then add 10 ml concentrated sulfuric acid.

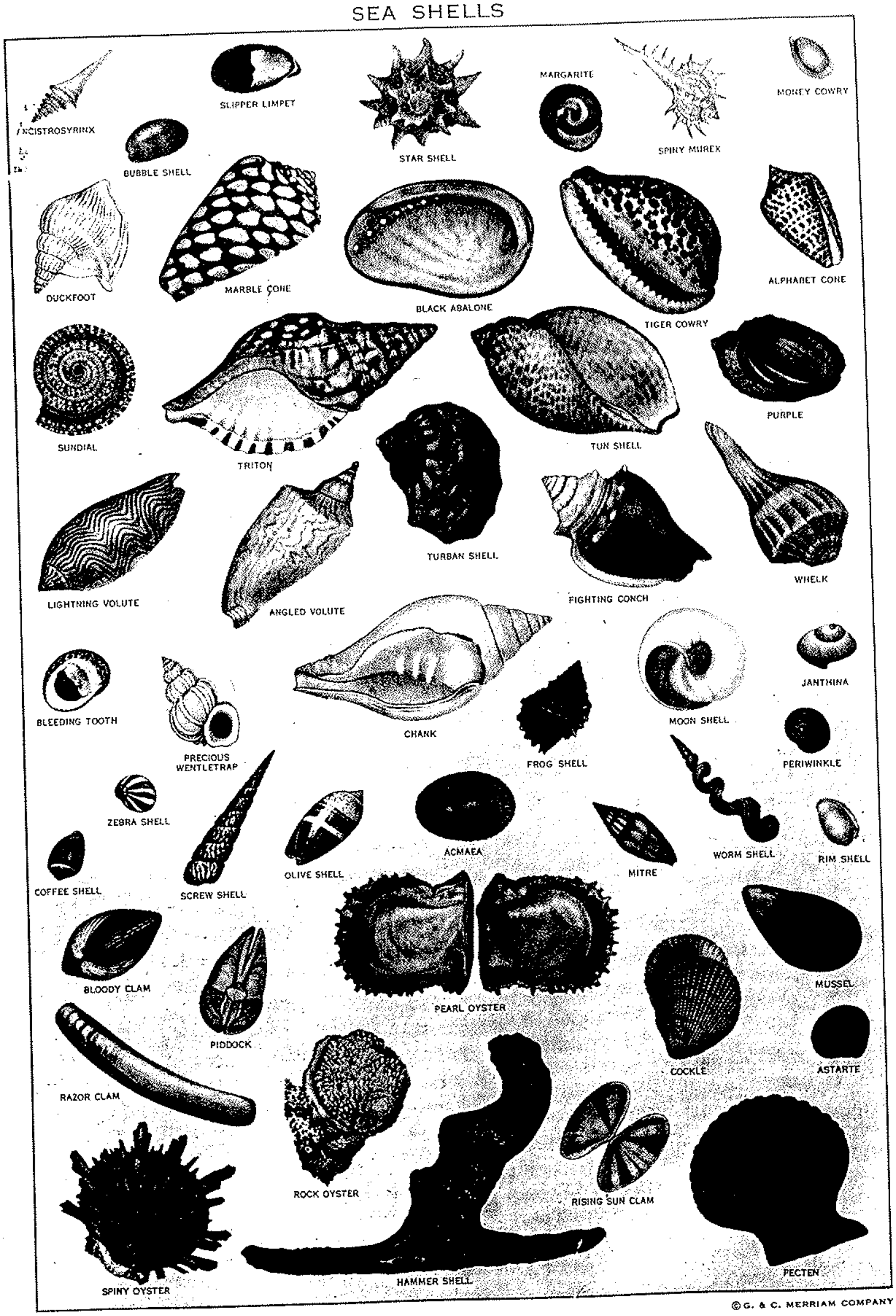
SPIRALS & HELICES: Spirals are mathematical forms which appear in many facets of nature, such as the curve of a fiddlehead fern, vines, shells, tornadoes, hurricanes, pine cones, the Milky Way, whirlpools. There are flat spirals, three dimensional spirals, right and left handed spirals, equiangular, logarithmic, hyperbolic, Archimedean spirals, and helices are just some of the many types of spirals which mathematics describes. The equiangular spiral appears in such growth forms of nature as the nautilus shell, a sunflower seedhead, the webs of Orb spiders. Some of the properties of the equiangular spiral are—angles formed from tangents to the spiral's radii are congruent (hence the term equiangular) — it increases at a geometric rate, thereby any radius is cut by the spiral into sections that form a geometric progression — its shape remains the same as it grows.



The elegant spirals of slime mould amoebas.



SEA SHELLS



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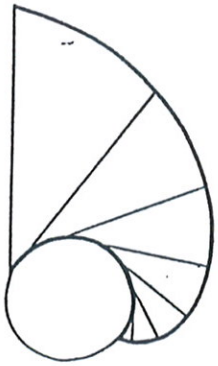
INVOLUTE CURVE: As a rope is wound or unwound around another curve (here a circle), it describe an involute curve. Involute is the shape found in the beak of an eagle, the dorsal fin of a shark, and the tip of a hanging palm leaf.



B) To 10 g of sodium bromide add water up to 100 ml.

C) To 10 g malonic acid add water up to 100 ml.

Now, pour 6 ml of solution A in a glass beaker and add .05 ml of solution B. Quickly mix in 1 ml of solution C. Place this brown mixture by an open window until it turns pale yellow to clear. Add 1 ml of 1,10-phenanthroline ferrous complex and mix thoroughly. Pour this solution into a Petri dish on a white background and you will see it first turn patchy blue, and then turn clear to brownish-red. In about five minutes blue dots will appear and grow into a series of slowly expanding concentric rings of blue and orange. Now, if you gently scrape a car key through this pattern of rings you will create spirals. This spontaneous spiraling effect lasts about twenty minutes.

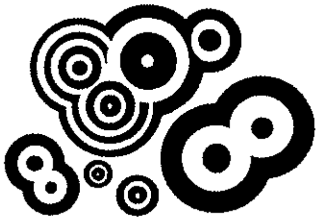
If you do not have the chemicals to perform this experiment you can easily find spiraling motions in many other liquids. The next time you take a bubble bath or add cream to coffee, keep an eye out for spirals.

Rising and Falling Spirals

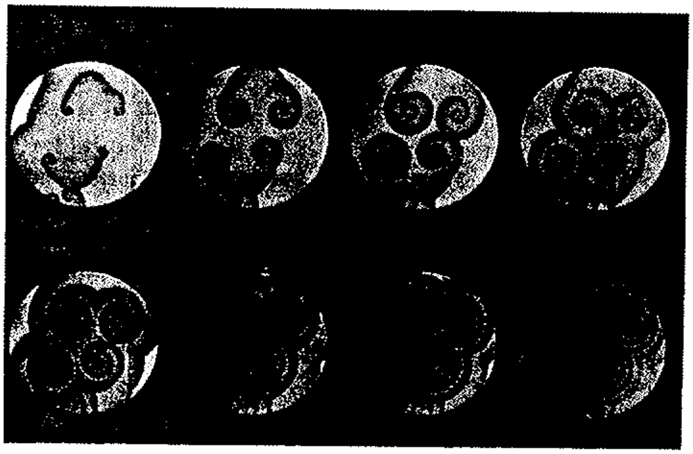
Light some incense and place it in an incense burner. Watch the smoke rise. You will see spirals, along with other swirling shapes, as the smoke interacts with the air currents. When you pray you can imagine that your prayers to God are like this incense smoke, rising upwards, dancing and spiraling all the way to heaven.

Like the Psalm! What a fascinating concept that our prayers would be [illegible] with God's pattern – reflecting it back!

Get an aluminum pie pan. Use a nail to poke a hole through the center of the pan. You will then need to make the hole a little bigger, so put a pencil through the hole to enlarge



The expanding rings of the BZ reaction.



Spirals in the BZ reaction.

it. Now on the bottom of the pan tape the hole over with masking tape. Next, put some liquid soap in the pan and then fill the pan with water. The soap suds will allow you to better see the motion patterns of the water. Hold the pan over a sink, remove the tape from the hole, and watch what happens. As the water drains out, you will see a small whirlpool form as the water spirals down the hole. Imagine that the water in the pan is our universe and that the hole is Christ. Christians believe that everyone and everything in our universe is spiraling towards union with Christ, the Omega point of evolution and our final destiny.

wonderful!

Our spiral God

God is the greatest of mysteries and can be thought of in many different ways. One way to think of God is as a great spiral that connects and nurtures all life. God grows like an equiangular spiral grows. Remember how we learned that an equiangular spiral retains its same shape while all the while growing larger. You can think of God like this, ever expanding but ever staying the same. The connection between God and the spiral is very old. The Pythagoreans of ancient Greece viewed the presence of the golden spiral in nature as proof for the existence of a divine creator who designed the cosmos according to the laws of sacred geometry. The ancient Celts made use of the spiral as their symbol for the Earth Goddess, as the spiral aptly represents the creative fertility of the Great Mother of Life. A great Persian sage of long ago named Zoroaster conceived of the God of Light as having the body of a spiral and the head of a hawk. Can you picture that?

The spiral journey to God

The path to God is a spiral path, leading one through on-going cycles of inner silence and outer action. In Zen Buddhism the spiritual journey to enlightenment is seen as a repeating cycle of ten stages, represented by the famous Zen ox-herding pictures. In the Gospels of Christianity we see Jesus' spiritual journey in which he moves from solitary prayer in the wilderness to marketplace activism over and over again.

Another interesting spiritual spiraling can be seen in the Genesis account of creation in which God creates the world in a six day cycle. On the seventh day, the Sabbath, God rests, and the entire circle of creation is lifted up to God. By honoring the Sabbath we open up our circle of time to the timeless Creator and each passing week becomes another spiral turn leading us closer to the God of life and mystery.

The Genesis story of the Garden of Eden actually contains a hidden golden spiral. To discover this spiral you must first understand that the original language of the story, Hebrew, is an alpha-numeric language in which every letter also represents a number. Gen. 2:8-9 states that Eden was located in the east and that in the center of Eden was the tree of life. The Hebrew word for "east", qedem, can also mean "eternal", and has a value of 144. The Hebrew phrase for "tree of life", eitz chayim, equals 233. Tell me this is not strange: 144 and 233 are consecutive Fibonacci numbers and 233/144 = 1.618!

Kabbalistic thinking: I like it.

We have been speaking of "the golden mean" as a mathematical value (1.618), but it is also a philosophical concept. Aristotle first developed the philosophy of the golden mean, which can be stated as follows: in order to attain happiness one must always act in moderation, striving to maintain the mean between the two extremes of excess and deficiency. All virtues according to Aristotle are virtues of moderation. The virtue of courage, for instance, is the mean between cowardice and foolhardy rashness. Our moral virtue, in this view, depends upon our conformity to the golden mean. Do you agree? Choose some virtue, such as patience or charity, and try to determine which two extremes this virtue lies midway between.

Final Spiraling Thoughts

What do you think it means that our entire cosmos is filled with spiral patterns? Having studied and mediated upon these spiral mysteries, what can we conclude? We can truly say that we are life forms composed of spirals, marked with spirals, living in a spiral, moving in a spiral, growing in a spiral, making a spiral spiritual pilgrimage, interconnected and nurtured by our ever spiraling God. Let us therefore always honor the principles of life which are symbolized by the spiral: creativity, diversity, evolution, virtue, balance, beauty and harmony. Spiral on!

> "My new hypothesis: If we're built from spirals while living in a giant spiral, then everything we put our hands to is infused with the spiral" (Aronofsky).

> "[E]nlightenment is alive, like a tree" (Nhat Hanh 137).

> "And consciousness grows in spirals" (Kornfield 68).

David, I've spent a lot of time thinking about spirals too. You've taken me even further in my thinking. Thank you.

an incredible paper! You should publish this

The following table of Aristotle's virtues and vices has been adapted from a similar table by the well-known Aristotelian scholar, W. D. Ross*.

FEELING	ACTION	EXCESS VICE	VIRTUE	DEFECT VICE
1. fear		cowardice	courage	none
confidence		rashness	courage	cowardice
2. pleasure, or bodily desire (mainly touch)		debauchery	self-control	insensibility
3. anger		short temper	gentleness	apathy
Practical Activity	1. giving of money	extravagance	generosity	stinginess
	taking of money	stinginess	generosity	extravagance
	2. spending of money on a grand scale	vulgarity	magnificence	niggardliness
	3. pursuit of honor on a grand scale	vanity	high-mindedness (self respect)	humility
	4. pursuit of honor on a small scale	ambition	none	lack of ambition
Life in Society	1. behavior in social relations	obsequiousness (flattery)	friendliness	grouchiness
	2. behavior for relaxation	buffoonery	wittiness charm	boorishness
	3. self-appraisal	boastfulness	truthfulness	self-depreciation
Mean states of feeling	1. shame	bashfulness	modesty	shamelessness
	2. pain at others' good or bad fortune	envy	righteous indignation	ill will

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